THE FUTURE OF High School Mathematics

New Priorities and New Resources
Whether you are a parent or a politician, whether you work in business, industry, government or academia, the state of U.S. MATHEMATICS EDUCATION is of fundamental importance to you and those you care about. As the importance of mathematical and quantitative thinking increases, we must become more focused as a nation on providing our children a better mathematical education.

~ From the Mission Statement of Math Is More, 2008
October 4, 2007 was the fiftieth anniversary of the Sputnik launch that led immediately to deep concern about the quality of U.S. mathematics education and recommendations for reform. Not surprisingly, reactions to Sputnik focused on the importance of mathematics education to national defense and on the need to prepare our most capable high school students for careers in mathematics, science, and engineering. Just over 25 years later, with the 1983 release of the Nation at Risk and Educating Americans for the 21st Century reports, the rationale for reform of mathematics education had changed to focus on its importance for international economic competitiveness—the concern that has persisted to the present day.

Today’s high school students need to be prepared for life in a world where responsibilities of advanced study, work, and citizenship require mathematical understanding and skill far different from what was acceptable only a short time ago. On a daily basis we use statistics to make election forecasts, environmental policies, and decisions about our personal health. Important principles from algebra and geometry are basic to the design of medical treatments like laser eye surgery and diagnostic devices like CAT scans and MRI. We use calculators and computers to manage large and small businesses and our personal finances. Rapid and effective access to resources on the World Wide Web is made possible by its mathematical design. Careers in familiar areas of the sciences, engineering, and business require broader and deeper mathematical knowledge than ever before, and emerging fields like computer science and operations research are opening brand new aspects of the subject.

Traditional approaches to mathematics curricula, teaching, and assessment will not meet the challenge of preparing more students for the demands of a changing world. Too many adults have unpleasant memories of their high school mathematics, and they avoid using mathematical reasoning even when it would be useful. For far too long, many students have been relegated to mathematics courses that fail to prepare them for future study and work. As a result, there are unacceptable achievement gaps between students from different economic, social, and cultural sectors of society.

Every student—not just a select few—has the right to be mathematically prepared for the future.

A new workforce of problem-solvers, innovators, and inventors who are self-reliant and able to think logically is one of the critical foundations that drive innovative capacity in a state. A key to developing these skills is strengthening science, technology, engineering, and math (STEM) competencies in every K-12 student.

Building a Science, Technology and Math Agenda
National Governors Association, February, 2007, p. 1
IN RESPONSE TO THE CHALLENGES AND OPPORTUNITIES for mathematics education in an increasingly quantitative society, mathematics educators have been working hard to develop and apply new kinds of curriculum materials, teaching strategies, and tools for assessing student knowledge. Following the publication of NCTM guidelines for school mathematics, five major NSF-funded curriculum projects and several other projects funded by private foundation grants developed and field-tested materials to support high school mathematics instruction that reflects the recommendations and goals of those advisory standards.

At the same time, innovative curriculum projects focused on middle school mathematics developed materials that provide students with enhanced preparation for success in high school. Activities at the collegiate level—the MAA Curriculum Foundations project, the MAA CUPM Curriculum Guide, the various calculus reform curriculum materials projects, and the AMATYC Crossroads recommendations—encouraged reconsideration of the mathematics curriculum content that students should master by the time they leave high school and the ways that subsequent collegiate mathematics can be taught most effectively.

In the past eight years, concern about the condition of high school mathematics has been expressed with renewed urgency. Reports from the Glenn Commission (Before It's Too Late), the National Commission on Prospering in the Global Economy of the 21st Century (Rising Above the Gathering Storm), the American Diploma Project, the College Board Mathematics and Statistics Advisory Committee, the American Statistical Association GAISE project, the President's National Mathematics Panel, the NCTM Principles and Standards for School Mathematics and High School Mathematics Curriculum Project, and the Carnegie-IAS Commission on Mathematics and Science Education have all made the case for transformation of curriculum, teaching, assessment, and teacher professional development.

Concurrent with those spirited national discussions about the future of school and collegiate mathematics, the most widely used innovative curricula at the high school level have completed major revisions of their programs, to reflect learning from use of first edition student and teacher materials. Those revised curriculum materials are complemented by emergence of other new approaches to high school mathematics. Each project has based its work on findings from contemporary research on mathematics teaching and learning and on thoughtful analysis of the mathematics likely to be most useful for students in their future studies and careers. The instructional materials provide solid foundations in basic mathematical skills, but they also build students’ abilities to solve challenging problems and to learn new mathematical ideas and skills in the future. In each project, experimental versions of the materials for students and teachers were tested extensively and then revised, to make sure that the final products work as intended in the full range of American schools. Extensive resources for teachers accompany each new program.

The concerns of national policy relating to mathematics education go far beyond those in our society who will become scientists or engineers. The national work-force of future years will surely have to handle quantitative concepts more fully and more deftly than at present. So will the citizens and policy leaders who deal with the public interest in positions of civic leadership. Sound education in mathematics across the population is a national interest.

NATIONAL DIALOGUE ABOUT
Action Priorities

TEACHERS AND LEADERS OF SCHOOL MATHEMATICS PROGRAMS face the challenge of choosing among competing proposals for curriculum content, teaching approach, and assessment strategies and the subsequent task of effectively implementing change. To introduce the results of recent theoretical and development work to key mathematics educators and to place those developments in a broader context of mathematics education innovation, the Math Is More group, a coalition of curriculum development projects, and the University of Maryland Center for Mathematics Education organized a national conference on The Future of High School Mathematics to demonstrate and analyze progressive ideas about curriculum, teaching, assessment, and technology in high school and early college mathematics. The plenary and workshop sessions of the conference focused on three central questions:

• What are the most important mathematical concepts, skills, and reasoning methods that students of different interests and aptitudes should master in the high school years so that they are well prepared for college, the world of work, and effective citizenship in the 21st century?
• What instructional practices hold greatest promise for effective teaching of mathematics to the diverse student population in U.S. high schools?
• What practices in assessment of student understanding and skills most effectively advance teaching and learning and provide an evidence base for important educational policy decisions?

The conference registered a capacity crowd of 260 professionals, from all over the country and representing all sectors of the mathematics education enterprise—classroom teachers, leaders of local and state school mathematics programs, mathematicians, teacher educators, education researchers, government officials, and developers of commercial textbooks, assessments, and software for mathematics education. They listened with interest to plenary talks addressing the core conference questions. Remarks by mathematician David Mumford, teacher educator and education policy expert Suzanne Wilson, National Council of Teachers of Mathematics past president Cathy Seeley, Mathematical Association of America president David Bressoud, and innovative publisher Steve Rasmussen described promising future directions for mathematics, challenges and strategies for shaping mathematics education policy, and prospects for radical new developments in the instructional materials and technology resources that support high school mathematics curricula and teaching. Video recordings of those plenary talks are available for viewing at the Math Is More website (www.mathismore.net).

In a series of extended breakout sessions, conference participants shared their personal views about principles that should guide progressive action to improve high school mathematics curricula, teaching, and assessment. Then another set of breakout sessions allowed participants to learn about and examine critically an array of innovative high school mathematics programs that have been developed and tested recently.

To achieve success in college, the workplace and life, American students must not only master important content, they must also be adept problem-solvers and critical thinkers who can contribute and apply their knowledge and skills in novel contexts and unforeseen situations...

High school graduates must also be able to work collegially in teams and be keenly aware of the rapidly changing world around them.

Out of Many, One: Toward Rigorous Common Core Standards from the Ground Up
Achieve, 2008. p. 5
The mathematics students need to learn today is not the same mathematics that their parents and grandparents needed to learn. When today’s students become adults, they will face new demands for mathematical proficiency that school mathematics should attempt to anticipate. Moreover, mathematics is a realm no longer restricted to a select few. All young Americans must learn to think mathematically, and they must think mathematically to learn.

Adding it Up: Helping Children Learn Mathematics
National Research Council, 2001, Executive Summary

Charting A Course of Action
Following the conference deliberations, participants were invited to respond to questions of an on-line survey that sought a sense of the group about priorities for action to improve high school mathematics. In each area—curriculum content, teaching, and assessment—the questions asked participants to rate the importance of various proposals for change in current practice. Over 60% of the participants responded to the specific priority questions, and many added spirited elaborating remarks about the issues and their personal concerns. Taken as a whole, the responses constitute a strong endorsement of the need for striking systemic change in the goals and practices of high school mathematics and they outline a coherent direction for action to implement needed change.

New Directions for Curriculum Content
The rapid mathematization of work in almost all areas of business, industry, personal decision-making, and the social and life sciences has made the case that all high school students need to learn more and possibly different mathematics than what traditional curricula provide.

To meet this challenge, over 95% of survey respondents assigned at least high or medium priority to actions that would implement explicit and pervasive emphasis on problem solving, authentic and diverse applications, and productive habits of mathematical thinking. Similar numbers endorsed movement toward a curriculum that develops key ideas from the broad array of mathematical sciences—including probability, statistics, and discrete mathematics as well as algebra, geometry, and calculus—and they agreed that students must be prepared to use computing tools effectively in exploration of mathematical ideas and solution of problems.

New Directions for Teaching Practice
Reflecting widespread belief in the proposition that what you learn depends in important ways on how you are taught, the history of mathematics education has included a persistent strand of proposals to change the “standard paradigm” of U. S. mathematics teaching.

Nearly 90% of survey respondents gave high or medium priority to actions that would transform the traditional standard of instructional practice to make independent, collaborative, and whole-class work on significant mathematical and applied problems the predominant activity in high school mathematics classes. Similar numbers endorsed the propositions that calculators and computers should be a regular presence in the high school mathematics classroom and that aspects of mathematical thinking like strategic competence, adaptive reasoning, productive disposition, and communication of ideas should be also be explicit foci of classroom activity and discussion.

New Directions for Assessment of Results
The best laid plans for transformation of curriculum content and teaching will make little headway in schools unless success of innovations is evaluated by assessments that are aligned with the new goals.

Over 85% of survey respondents assigned high or medium priority to assuring that high stakes assessments measure student ability to solve significant mathematical and applied problems and to produce the kinds of constructed response solutions...
that would be expected in real-life situations. They also endorsed the proposition that aspects of mathematical thinking and communication should be explicit foci of assessment and that greater emphasis should be placed on using assessment as a tool for instruction that builds on students’ prior knowledge and addresses their learning needs.

The only item on the entire survey that garnered less than a majority high priority response was the proposition that “Assessment of students’ mathematical understanding and skill should be designed with assumptions of access to calculators and computers.” As with items about technology effects on curriculum content and teaching, conference participants were ambivalent, but still generally optimistic, about the role of calculators and computers in shaping the high school mathematics experience.

Resources and An Agenda for Action

Taken all together, the presentations, breakout discussions, demonstrations of innovative curriculum projects, and responses to the post-conference survey present a picture of enthusiastic support for a coherent set of initiatives to improve the content, teaching, and assessment practices of high school mathematics. They also suggest the existence of leaders eager to undertake the work required to make those initiatives the new standard of practice in the field. Significant numbers of conference participants reported interest in being involved with research and development projects designed to operationalize and test the various reform ideas, with the greatest number showing enthusiasm for work on professional development with teachers.

We live in a time of extraordinary and accelerating change. New knowledge, tools, and ways of doing and communicating mathematics continue to emerge and evolve. The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase.

*Principles and Standards for School Mathematics*
NCTM, 2000, Executive Summary

CHOOSING AMONG New Resources

The following sections of this document provide overviews of student and teacher resources produced by seven prominent innovative high school mathematics curriculum projects that were demonstrated and analyzed at the conference. Each overview includes description of the project goals and accomplishments and excerpts that illustrate the style of materials provided.

<table>
<thead>
<tr>
<th>Page</th>
<th>Project Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Page 6</td>
<td>COMAP – Mathematics: Modeling Our World</td>
</tr>
<tr>
<td>Page 8</td>
<td>CME Project at the Education Development Center</td>
</tr>
<tr>
<td>Page 10</td>
<td>Core-Plus Mathematics Project</td>
</tr>
<tr>
<td>Page 12</td>
<td>Interactive Mathematics Program</td>
</tr>
<tr>
<td>Page 14</td>
<td>MATH Connections</td>
</tr>
<tr>
<td>Page 16</td>
<td>SIMMS Integrated Mathematics</td>
</tr>
<tr>
<td>Page 18</td>
<td>University of Chicago School Mathematics Project</td>
</tr>
</tbody>
</table>

Guidelines for selection and implementation of these mathematics programs and a complete listing of project contact information are provided on pages 20–21.
For 30 years, COMAP (Consortium for Mathematics and its Applications) has been dedicated to presenting mathematics through contemporary applications. We have produced high school and college texts, hundreds of supplemental modules, and three television courses—all with the purpose of showing students how mathematics is used in their daily lives.

In the COMAP spirit, *Mathematics: Modeling Our World* demonstrates mathematical concepts in the contexts in which they are actually used. The word “modeling” is the key. Real problems do not come at the end of textbook chapters. Real problems don’t look like math problems. Real problems are messy. They ask questions such as, How do we create a computer animation? How do we effectively control an animal population? What is the best location for a fire station? What do we mean by “best”?

Mathematical modeling is the process of looking at a problem, finding a mathematical core, working within that core, and coming back to see what mathematics tells us about the original problem. We do not know in advance what mathematics to apply. The mathematics we settle on may be a mix of geometry, algebra, trigonometry, data analysis and probability.

We may need to use computers or graphing calculators, spreadsheet, or other utilities. Because *Mathematics: Modeling Our World* brings to bear so many different mathematical ideas and technologies, our approach is truly integrated.

At heart, we want to demonstrate to students that mathematics is the most useful subject they will learn. Most importantly, we hope to demonstrate that using mathematics to solve interesting problems about how our world works can be a truly enjoyable and rewarding experience.

*Mathematics: Modeling Our World* is available from:

175 Middlesex Turnpike, Suite 3B
Bedford, MA 01730
800-772-6627
Fax: 781-863-1202
Email: info@comap.com
Web: www.comap.com
Course 1
Unit 1  Pick A Winner: Decision Making in a Democracy
Unit 2  Secret Codes and the Power of Algebra
Unit 3  Scene from Above
Unit 4  Prediction
Unit 5  Animation/Special Effects
Unit 6  Wildlife
Unit 7  Imperfect Testing
Unit 8  Testing 1, 2, 3

Course 2
Unit 1  Gridville
Unit 2  Strategies
Unit 3  Hidden Connections
Unit 4  The Right Stuff
Unit 5  Proximity
Unit 6  Growth
Unit 7  Motion

Course 3
Unit 1  The Geometry of Art
Unit 2  Fairness and Apportionment
Unit 3  Sampling
Unit 4  Mind Your Own Business
Unit 5  Oscillation
Unit 6  Feedback
Unit 7  Modeling Your World

Course 4
Chapter 1  Functions in Modeling
Chapter 2  The Exponential and Logarithmic Functions
Chapter 3  Polynomial Models
Chapter 4  Coordinate Systems and Vectors
Chapter 5  Matrices
Chapter 6  Analytic Geometry
Chapter 7  Counting and the Binomial Theorem
Chapter 8  Modeling Change with Discrete Dynamical Systems

Mathematics plays an important role in many aspects of your life. That’s why you have studied it since you were a child. In this unit, the percentages, decimals, and calculations with which you are already familiar will be used in ways that are probably new to you. You will see new mathematical representations that help you better understand problems and find better solutions. You will learn more about the way mathematics is used in the world.

Although mathematics is used to solve problems, it does not always give perfect solutions to real-world problems. When solutions to problems are not perfect, mathematics can be used to improve them. In this work, you will see that—just as you will use mathematics to understand why solutions are not perfect, you will also use mathematics to improve on those solutions. You will do so by examining the problem that is your responsibility in your society.

You live in a democracy. As a United States citizen, when you reach the age of 18 you will have the right to vote. In your lifetime you will vote for candidates for school boards, governor, senator, president, and many others. All of the other citizens of this country have the same right. However, they do not all feel the same way about the candidates and issues on which you vote. In order for the country, states, and cities to function, the votes of all citizens must be combined to produce a single decision on who will lead the nation, your state, and your city. Therefore, the central question of this unit is: How does a group make a democratic decision?

Voting has been shown to be the topic of the first unit in this textbook because it is an experience shared by everyone. Although this unit is about voting, it is also about mathematics and the way mathematics is used to solve real-world problems. Mathematics is used to determine the winner whenever people vote. The mathematics is often simple, but it does an important job by selecting the people who run our governments, the products that are in our stores, and the programs that are on our televisions.
The CME Project is a comprehensive four-year high school mathematics program developed by the Center for Mathematics Education (CME) at Education Development Center, Inc. (EDC) and funded by the National Science Foundation. The problem-based, student-centered program offers an innovative curriculum organized around the familiar themes of Algebra 1, Geometry, Algebra 2, and Precalculus. The CME Project sees these branches of mathematics not only as compartments for certain kinds of results, but also as descriptors for methods and approaches—the habits of mind that determine how knowledge is organized and generated within mathematics itself.

The CME Project sets as its goal mathematical rigor and accessibility for all students by emphasizing the interplay between mathematical thinking and essential technical skills.

Core Principles of the CME Project

- **Habits of mind.** CME Project focuses on developing students’ habits of mind. The familiar course structure allows students time and focus to develop algebraic, geometric, and analytic habits of mind.

- **A coherent curriculum.** The program draws on lessons learned from high-performing countries: ideas are developed thoroughly and revisited only to deepen them and their connection with other ideas, omitting the clutter of extraneous topics.

- **Experience before formality.** CME Project employs the best American models that call for grappling with ideas and problems as preparation for instruction, moving from concrete problems to abstractions and general theories.

- **Textured emphasis.** CME Project separates convention and vocabulary from matters of mathematical substance. Even practice problems are designed to have a larger mathematical point.

- **General purpose tools.** Fluency in algebraic calculation, proof, and graphing are essential in mathematics, but special-purpose techniques, such as FOIL and the rote application of $y = mx + b$, are not.

- **High expectations.** Students at all levels have the capacity to think in ways that are characteristically mathematical. Field tests have demonstrated students of all abilities and backgrounds can meet these expectations.

Development of CME Project

CME Project writers, field-testers, reviewers, and advisors come from all parts of the mathematics community: teachers, mathematicians, education researchers, technology developers, administrators, and students. During the development phase, all CME Project materials were extensively field-tested in diverse classrooms across the country. Independent research conducted in each year of field-testing showed that the CME Project can be successful in raising student achievement.

“I teach in an included classroom, with both regular education and special education materials, in an inner city public high school. I was thrilled to be able to present the [CME] materials to all of my students and to have all of the students engaged. ...Needless to say, I was excited to teach with these materials.”

– James Stallworth
Cincinnati, Ohio

“I got so excited, the students were very concerned! I've never had a student come up with how to use slope-intercept form to find the equation of a line before-all I've gotten [were] blank stares!”

– Annette Roskam
Rice Lake, Wisconsin
A SAMPLE FROM CME PROJECT

CME Project uses innovative methods, developed over decades of teaching experience, that help students master difficult topics, such as algebra word problems, graphing, function algebra, complex numbers, proof, and data analysis.

GETTING STARTED PROBLEM

This Getting Started problem from Algebra 1 Chapter 4, Lines, introduces the concept of slope.

Getting Started

With the CME Project materials, students launch into each new topic with a “Getting Started” lesson that activates prior knowledge and explores new ideas. This provides students the opportunity to grapple with ideas and problems before these concepts are brought to closure and formalized.

Minds in Action

Discussing mathematical understandings is an effective vehicle for student learning. By featuring dialogues between recurring characters, the CME Project exposes students to different ways of communicating about mathematics.

In this Minds in Action from Geometry, Chapter 6, Sasha and Tony discover a theorem that uses similarity to prove the concurrence of the medians in a triangle.

Developing Habits of Mind

Methods and approaches, or mathematical “Habits of Mind” are developed and practiced throughout the text. They include visualization, performing thought experiments, reasoning by continuity or linearity, and mixing deduction with experiment.

This is a typical example of a feature called “Developing Habits of Mind” from Algebra 2.
The Core-Plus Mathematics Project (CPMP) has developed, field-tested, and evaluated student and teacher curriculum materials for high school mathematics that make important and broadly useful mathematics meaningful and accessible to a wide range of students. The curriculum consists of a single core sequence for both college-bound and employment-bound students in the first three courses. The fourth course consists of units to continue the preparation of students for college mathematics.

Key Features of the Curriculum Materials Include:
- Development of mathematics as an active science of patterns involving quantity and change, shape and motion, data and chance, and counting and algorithms;
- Integrated development of fundamental concepts and skills in algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics;
- Student-centered investigations that promote active learning through problem solving;
- Mathematics developed in context with an emphasis on applications and mathematical modeling; and
- Full and appropriate use of technology tools, including CPMP-Tools® software for each course.

With funding from the National Science Foundation, the project has just completed a major update and revision of its texts, teacher materials, and software. Core-Plus Mathematics second edition builds on the strengths of the first edition that was cited as Exemplary by the U.S. Department of Education.
High Punkin’ Chunkin’ Compressed-air cannons, medieval catapults, and whirling slings are used for the punkin’ chunkin’ competitions. The pumpkin height at any time $t$ will depend on its speed and height when it leaves the cannon.

Suppose a pumpkin is fired straight upward from the barrel of a compressed-air cannon at a point 20 feet above the ground, at a speed of 90 feet per second (about 60 miles per hour).

a. If there were no gravitational force pulling the pumpkin back toward the ground, how would the pumpkin’s height above the ground change as time passes?

b. What function rule would relate height above the ground $h$ in feet to time in the air $t$ in seconds?

Each investigation culminates with summary questions for class discussion to articulate the key mathematical ideas of the investigation.

**Student Achievement**

CPMP work has included an extensive component of formative and summative evaluation. Extensive field-testing of both the first and second editions and the longitudinal study have provided valuable information to inform the published *Core-Plus Mathematics* materials. In addition, individual school districts using the program have monitored and reported student achievement for their populations. Evaluation studies related to *Core-Plus Mathematics* are available at [www.wmich.edu/cpmp/evaluation.html](http://www.wmich.edu/cpmp/evaluation.html). Check this site for a forthcoming publication containing comprehensive results of the longitudinal study and new school reports.

Results from evaluation of the second edition indicate that CPMP students outperformed comparison groups on the ETS Algebra test, the ACT, and college placement tests.

A consistent finding by districts is increased enrollment in Advanced Placement Calculus and Statistics courses and increases in the percentage of students gaining credit in these courses.

**Core-Plus Mathematics** Teacher Quotes:

“I know that the kids are learning more, they remember it longer, and they put all the pieces together to form the big picture (the connections are everywhere).”

“The jump in difficulty level each year always amazed me. I would not have believed that such a variety of students could be successful in this curriculum.”

**Core-Plus Mathematics** Student Quotes:

“In the past, if a problem was too hard, I would simply skip it. Now, I can look at a problem, think about it, and then, pick it apart until I find a solution.”

“The real-life examples of Core-Plus Mathematics gave me an excellent background for demanding engineering courses. Because of my Core-Plus background, I feel I am two steps ahead of students who did not take Core-Plus Math in high school.”
IMP Promotes Understanding
The Interactive Mathematics Program (IMP) was conceived nearly twenty years ago when the state of California issued a request for proposals for revamping the traditional sequence of high school mathematics. In place of Algebra I – Geometry – Algebra II, the proposal guidelines called for a modern mathematics curriculum with problem solving, reasoning, and communication as major goals. The new curriculum would also include such areas as statistics and discrete mathematics; it had to be flexible enough to meet the needs of all college-bound students and lower the attrition rate of students in the college prep sequence, especially women and minority students.

IMP Provides a Problem-Solving Curriculum
The IMP curriculum is organized by units, each of which generally begins with a central problem to be explored and solved over the course of six to eight weeks. As students work through a variety of smaller problems within the unit, they develop the mathematical concepts and techniques they need to solve the central problem. Some central problems are based in practical, real-world situations—how to maximize profits for a business, for instance—while others are more fanciful, exploring such situations as a pennant race or a circus act; many central problems have connections with history, science, or literature. The problem focus of the curriculum gives students a rich look at how mathematics is actually used—a feature that is often lacking in traditional textbooks.

IMP lets students be active, engaged learners, using what they already know, making conjectures, and learning from errors. The curriculum presents students with rich mathematical contexts and gives them well-designed opportunities to discover and develop mathematical concepts as well as to prove important results. This approach offers students meaning for abstract concepts, gives them ownership of mathematical ideas, and heightens their interest in mathematics.

The curriculum includes the fundamental ideas that have long been part of the high school syllabus—concepts and skills from algebra, geometry, trigonometry, and calculus. Although this material is organized in a new way and embedded in problem-based units, the key ideas are all there and students regularly use and revisit them. Each unit usually combines several branches of mathematics so that students see how important ideas are related to each other. In addition to the traditional material, students also learn about such branches of mathematics as statistics or matrix algebra, which are new to the high school curriculum but are used throughout business and industry today.

Developed collaboratively by mathematicians, teachers, and teacher educators, IMP’s four-year core curriculum prepares students for college and gives them problem-solving skills they will need in higher education and on the job. The program was first published in 1997, following more than nine years of research, pilot tests, evaluation, field tests, revision, and detailed reviews by professionals in the field. The revision for the second edition began in 2002 and will continue until 2010. It too has undergone extensive review.
IMP Teacher's Guides Are Going Online

Comprehensive Teacher's Guides for all IMP units explain in detail how to present the material. Each guide gives a summary of the unit, describes how the unit is organized, and lists the main concepts and skills. It offers clear guidance on doing, discussing, and debriefing the activities, and it shows how the mathematical concepts should evolve from the activities. It gives specific hints teachers can offer or questions they can ask to promote student dialogue, and it provides additional mathematical background for teachers' reference.

The Teacher's Guide for all Year 1 units is now available online. Years 2, 3, and 4 Teacher's Guides are currently available in print and will be posted online in coming months. Each online guide covers the intent, mathematics, and progression for every activity, cluster of activities, and the unit as a whole. It includes examples of student work, mathematical discussions, sample lesson plans, and links to advice from experienced IMP teachers. The online guides are free. Future enhancements may include technology demonstrations and activities, classroom videos, and discussion forums.

IMP Is Effective: A Broad Range of Measures

IMP teachers enthusiastically report student success, and their personal observations are supported by hard facts. An extensive set of studies—including preliminary work during pilot testing, a comprehensive NSF-funded evaluation, and numerous independent studies—demonstrated these broad findings:

- IMP students consistently do at least as well as, and often better than, their peers in traditional programs on standardized tests.
- IMP students significantly outperform their peers in traditional programs on tests focused on quantitative reasoning, general problem solving, and statistics.
- IMP students demonstrate more positive attitudes about mathematics and take more mathematics courses, including advanced courses, than their peers in traditional programs.

Learn More about IMP

- Download the table of contents for all four years, get a detailed description of each unit, review extensive teaching resources, request examination materials, and more at www.keypress.com/imp.
- For information about IMP professional development, contact Teachers Development Group at www.teachersdg.org.
- For comprehensive IMP resources, visit the IMP online community at www.mathimp.org.
When Students Connect with Math, Achievement Increases

MATH Connections® began in 1992 with a five-year National Science Foundation grant awarded to the Education Foundation of the Connecticut Business and Industry Association (CBIA), whose interest in mathematics education stemmed from the inadequate mathematical skills of their members’ potential employees. As stated in the grant proposal, the overarching mission of the project was the development of a comprehensive three-year, core curriculum designed to serve the needs of all students and to make mathematics more appealing and more accessible to them. It was envisioned that this common, three-year core, would be followed by a choice of fourth-year options tailored to students’ differing educational and career goals, preparing them for postsecondary education or the workforce. A distinctive feature of the proposal was the planned use of the corporate resources of the member companies of the CBIA in developing the curriculum and in providing authentic applications from business and industry.

The principal investigators were Project Director June G. Ellis (CBIA Education Foundation), Robert Rosenbaum (Wesleyan University), and Robert Decker (University of Hartford). William Berlinghoff (Colby College), Clifford Sloyer (University of Delaware), Robert Hayden (Plymouth State College), and Eric Wood (University of Western Ontario) were the Senior Writers for the first edition, published by It’s About Time, Inc., of Armonk, New York. The assessment specialist was Don Hastings of the Stratford, Conn., public school system.

Senior Writers for the second edition were William Berlinghoff and Karen Santoro (Central Connecticut State University), working in conjunction with Barbara Zahm (the Project Director) and Monica Rodriguez, Jillian McKiernan and Matthew Buco of It’s About Time.

Goals

The curriculum is intended for all students, regardless of their future education or career plans. It is relevant to students as future citizens, parents, voters, consumers, researchers, employers, and employees. It serves the needs and interests of those who will go on to further mathematics-intensive studies and science-related careers, as well as those who choose to pursue other fields of study or enter the workforce directly after high school. This is reflected, in part, by a two-tiered approach to the content—an “inner core” of material considered essential for students of all ability levels and an “outer core” encompassing more challenging aspects of the topics covered.

It is flexible, affording students with different learning styles equal opportunity to master the ideas and skills presented. This means offering both interactive group work and individual learning experiences, encouraging frequent student-teacher and student-student interaction in finding answers and in discussing mathematics and students’ relevant experiences, opinions, and judgments.

It employs the main technological tools of the world of work. Students must learn to be comfortable with graphing calculators and computers in order to cope with and profit from the opportunities of our increasingly technological world.

It is reality-based. All the mathematical ideas in the curriculum are drawn from and connected to real-world situations, with care taken to avoid any oversimplifications that destroy the realism of the results. Students can immediately see how the mathematics they are learning relates to their own lives and the world around them.

It focuses on mathematical processes—reasoning, pattern-seeking, problem-solving, questioning, and communicating with precision. It is because those features of a mathematical education are as important to lawyers, doctors, business leaders, teachers, politicians, social workers, military officers, entrepreneurs, artists, and writers as they are to scientists or engineers.
As its name suggests, *MATH Connections* explores connections of all kinds: between mathematics and the real world of people, business, and everyday life; between mathematics and the sciences; between mathematics and other disciplines, including history, geography, language, and art; and among different areas of mathematics.

The content of each book has a unifying theme:

**Year 1 Data, Numbers, and Patterns**
**Year 2 Shapes in Space**
**Year 3 Mathematical Modeling**

Within those general themes, each chapter has its own unifying “story line.” The chapters are large (there are only 22 in the entire program), each one presenting a major topic in a way that fits into the theme of its book.

The four Chapters of the first book typify these organizational principles at work. The thematic goal, realized in Chapter 4, is the calculator-assisted use of linear regression to solve some real-world prediction problems. The rest of the book is organized around reaching this goal: single-variable statistical ideas in Chapter 1, first-degree equations in Chapter 2, and graphs of linear equations in Chapter 3. Practice in using a graphing calculator is built in along the way, so that by the time students reach chapter 4, they have all the tools needed to deal with the problems posed.

In research by an independent evaluator, high school students participating in *MATH Connections* were found to score significantly higher than non-participating students on the SATs two years in a row. In an evaluation of cognitive discourse levels, teachers who taught MATH Connections classes were found to evoke a broader range of reasoning, communicating and problem-solving skills in their students again significantly better than non-participating students.

On a statewide math test in Warwick, Rhode Island, over 1000 students at all three high schools showed significant increases in achievement since implementing *MATH Connections*. From 2002 to 2004, math skills increased 12%, math concepts increased 15%; and math problem-solving increased 21%.

The second edition will be published for Fall 2009. *MATH Connections* second edition follows the same goals and pedagogy as the first, with improvements based on feedback from teachers presently using *MATH Connections*. The new NCTM standards are addressed. Additional problem sets and reviews have been inserted at the end of each chapter. New graphics using conceptual illustrations connecting mathematics to the real world were added to engage and motivate students to increase student achievement.

Many of the topics from algebra, statistics, combinatorics, and probability introduced in Year 1 are revisited in a broader, deeper, more sophisticated way in Year 3. The first edition was published in 1998, and proved to show student success.
The Systemic Initiative for Montana Mathematics and Science (SIMMS) was a seven-year, cooperative initiative of the state of Montana and the National Science Foundation. One major project goal was to help students learn to make mathematically informed decisions through a complete redesign of the 9-12 mathematics curriculum using an integrated, interdisciplinary approach to reach all students. With this major goal was a direct effort to increase the participation of females and Native Americans in mathematics and science using the developed curriculum as one vehicle of reaching these audiences.

The very visible end product is a four year high school integrated mathematics curriculum that included all strands of the NCTM Curriculum and Evaluation Standards for School Mathematics (1989) and ultimately the NCTM Principles and Standards for School Mathematics (2002). Although the project began as a part of the systemic reform of mathematics in Montana, it became a nationwide curriculum with its publication by Kendall Hunt Publishing Company.

Results of the early field tests and preliminary studies on SIMMS Integrated Mathematics are included in Lott, et al (2003), and Lott and Souhrada (2000), Lott et al (2007). Souhrada (2001) conducted a four-year longitudinal study of students in one high school studying traditional mathematics curricula, the SIMMS Integrated Mathematics, and students studying a mix of these materials with research questions involving content knowledge, attitudes, and teaching methods used. He found no significant differences in PSAT scores of students even when students studying traditional curricula had one extra year of mathematics. On open-ended test items, students in the SIMMS curricula scored higher than traditional students all four years of the study but there was only a statistically significant difference in the fourth year.

The current curriculum includes the Student Edition and Teacher Edition for four Levels (years) of the curriculum, a Teacher’s Resource CD, an Assessment Handbook, and an Implementation Guide. A full range of professional development is available to assist in all aspects of implementation. More information is available at www.simms-im.com.
REFERENCES


Overall Project Goals and Underlying Philosophy
The University of Chicago School Mathematics Project (UCSMP), begun in 1983, is a multi-faceted project designed to improve the school mathematics experience for the vast majority of students in grades PreK-12, by upgrading student performance, updating the curriculum to include contemporary pure and applied mathematics and technology, and increasing the percent of students who take mathematics beyond algebra and geometry.

UCSMP has worked towards this goal by translating textbooks from other countries, holding international conferences, and developing curricula for all the grades with state-of-the-art comparative research on performance. The project is best-known for its *Everyday Mathematics* (EM) materials for grades K-6 and its secondary curriculum for grades 6-12.

Structural Underpinnings of the UCSMP

Grades 6-12 Curriculum

• A sound mathematics curriculum covers the mathematical sciences, including quantitative literacy and statistics, consumer mathematics, logic and discrete mathematics, algebra and functions, geometry and trigonometry, with attention to pure and applied (real-world applications and modeling) aspects of these areas. It prepares students for all the mathematics they might encounter in college.

• All students should encounter essentially the same mathematics curriculum, but not all students are ready for the same mathematics at the same age. (See chart.)

• Such curriculum cannot be covered in four years from algebra to calculus. Algebra can, should, and must be an 8th grade subject for students who have had a strong elementary school curriculum, who do daily homework, and who have qualified teachers. For better students, it should be a 7th grade subject, enabling calculus to be taken in the high school.

Instructional Features of the UCSMP Grades 9-12

Materials

• Students who have had previous UCSMP courses are at an advantage, but each course is designed to stand alone and to accommodate students who have not had any previous UCSMP experience.

• Courses are traditionally organized into chapters and lessons, with 100-110 lessons per book.

• Each chapter ends with a modified mastery-learning strategy: a self-test with feedback in the text, followed by a focused review organized by individual chapter objectives.

• The reading in the lessons includes perspectives on the content, explanations, activities, traditional examples, guided examples for students to complete, and quiz-yourself questions.

• The questions in each lesson are of four types: covering the basic ideas, applying and extending the mathematics, review, and exploration.

• The focus of explanations and performance is on building understanding of Skills (algorithms), Properties (mathematical underpinnings), Uses (applications, including mathematical modeling), and Representations (graphs and diagrams)—what we call the SPUR approach.

• Students are assumed to have calculators with graphing and DGS capability in all courses from Transition Mathematics on. Class sets of CAS calculators are assumed in UCSMP Algebra, and students are expected to have CAS-capable technology from UCSMP Advanced Algebra on.

Course Materials

UCSMP Algebra covers linear and quadratic expressions, sentences, and functions; and integrates geometry, probability, and statistics.

UCSMP Geometry utilizes coordinates and transformations throughout, and gives strong attention to measurement and three-dimensional figures. Work with proof writing follows a carefully sequenced development of the logical and conceptual precursors to proof.
UCSMP Advanced Algebra assumes and utilizes geometry throughout. It emphasizes facility with algebraic expressions and forms, including both the abstract and modeling properties of linear, quadratic, exponential, logarithm, trigonometric, polynomial, and other functions.

Functions, Statistics, and Trigonometry integrates statistics and function concepts. Enough trigonometry is available to constitute a standard precalculus course in the areas of trigonometry and circular functions.

Precalculus and Discrete Mathematics integrates the background students must have to be successful in calculus with the discrete mathematics helpful in computer science. It balances advanced work on functions and trigonometry, an introduction to limits, and other calculus ideas with work on combinatorics, recursion, and graph theory. Mathematical thinking, including specific attention to formal logic and proof, is a theme throughout.

Description of Field Tests, Evaluations, Research Evidence
Years of pilot studies, formative evaluations, and national summative evaluations are behind each text. UCSMP materials at all levels are among the most-tested materials available and the preponderance of evidence shows that students using UCSMP materials perform equally with comparable students on standardized measures and outperform them on content that goes beyond those measures.

### Availability of Materials and Teacher Training

UCSMP holds a secondary curriculum User’s Conference each year on the University campus. Go to [http://socialsciences.uchicago.edu/ucsmp/](http://socialsciences.uchicago.edu/ucsmp/) for more information. Wright Group also holds training conferences. In addition, both Wright Group and UCSMP have arranged for presentations at schools.

For more information:
University of Chicago School Mathematics Project
6030 S. Ellis Avenue
Chicago, IL 60637
Phone: 773-702-1130
E-mail: ucsmp@uchicago.edu
Web: [socialsciences.uchicago.edu/ucsmp/](http://socialsciences.uchicago.edu/ucsmp/)

<table>
<thead>
<tr>
<th>GRADE</th>
<th>TOP 10-20% OF STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>EM 6 or Pre-Transition Mathematics</td>
</tr>
<tr>
<td>6</td>
<td>Transition Mathematics</td>
</tr>
<tr>
<td>7</td>
<td>Algebra</td>
</tr>
<tr>
<td>8</td>
<td>Geometry</td>
</tr>
<tr>
<td>9</td>
<td>Advanced Algebra</td>
</tr>
<tr>
<td>10</td>
<td>Functions, Statistics, and Trigonometry</td>
</tr>
<tr>
<td>11</td>
<td>Precalculus and Discrete Mathematics</td>
</tr>
<tr>
<td>12</td>
<td>Calculus (Not available through UCSMP)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NEXT 50% OF STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM 6 or Pre-Transition Mathematics</td>
</tr>
<tr>
<td>Transition Mathematics</td>
</tr>
<tr>
<td>Algebra</td>
</tr>
<tr>
<td>Geometry</td>
</tr>
<tr>
<td>Advanced Algebra</td>
</tr>
<tr>
<td>Functions, Statistics, and Trigonometry</td>
</tr>
<tr>
<td>Precalculus and Discrete Mathematics</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NEXT 20% OF STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM 6 or Pre-Transition Mathematics</td>
</tr>
<tr>
<td>Transition Mathematics</td>
</tr>
<tr>
<td>Algebra</td>
</tr>
<tr>
<td>Geometry</td>
</tr>
<tr>
<td>Advanced Algebra</td>
</tr>
<tr>
<td>Functions, Statistics, and Trigonometry</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REMAINDER OF STUDENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>EM 6 or Pre-Transition Mathematics</td>
</tr>
<tr>
<td>Transition Mathematics</td>
</tr>
<tr>
<td>Algebra</td>
</tr>
<tr>
<td>Geometry</td>
</tr>
<tr>
<td>Advanced Algebra</td>
</tr>
<tr>
<td>Functions, Statistics, and Trigonometry</td>
</tr>
</tbody>
</table>

**UCSMP Advanced Algebra** assumes and utilizes geometry throughout. It emphasizes facility with algebraic expressions and forms, including both the abstract and modeling properties of linear, quadratic, exponential, logarithm, trigonometric, polynomial, and other functions.

**Functions, Statistics, and Trigonometry** integrates statistics and function concepts. Enough trigonometry is available to constitute a standard precalculus course in the areas of trigonometry and circular functions.

**Precalculus and Discrete Mathematics** integrates the background students must have to be successful in calculus with the discrete mathematics helpful in computer science. It balances advanced work on functions and trigonometry, an introduction to limits, and other calculus ideas with work on combinatorics, recursion, and graph theory. Mathematical thinking, including specific attention to formal logic and proof, is a theme throughout.

**Description of Field Tests, Evaluations, Research Evidence**
Years of pilot studies, formative evaluations, and national summative evaluations are behind each text. UCSMP materials at all levels are among the most-tested materials available and the preponderance of evidence shows that students using UCSMP materials perform equally with comparable students on standardized measures and outperform them on content that goes beyond those measures.
IMPLEMENTING NEW Mathematics Programs

THE HIGH SCHOOL MATHEMATICS PROGRAMS described in this document are currently being used by students in thousands of classrooms in urban, suburban, and rural schools across the country. The careful development and wide use of those curriculum materials has provided significant information about effective implementation strategies.

Plan for Long-Term Implementation
Because the innovative curricula represent substantial change in subject matter, teaching methods, and assessment tools used in high school mathematics, districts have found that a multi-year plan increases the likelihood of a smooth implementation. Though experience has shown that success comes in many forms, several common elements of such long-term plans have been identified.

- Before consideration of specific programs, teachers and key administrators should understand clearly the differences between new and traditional programs.
- With district goals in mind, mathematics program leaders should plan for an in-depth curriculum selection process involving parents, teachers, and administrators.
- School systems should consider pilot-testing several programs to compare their effectiveness in classrooms of your district schools.
- Teachers should plan for regular communication with parents about pending and emerging curriculum changes.

Professional Development and Teacher Support
Experience in implementing the new mathematics programs over the past several years suggests that a substantial, multi-year professional development plan is essential. In designing such long-term teacher support, school districts have learned that the needs of teachers change as implementation progresses.

- Before using a program, teachers need to become familiar with its goals, philosophy, structure, and management.
- Teachers report that during the first year of teaching a standards-based program, regular support through in-class help or networking makes a positive difference.
- Professional development activities should provide opportunities for teachers to extend their own mathematical knowledge and to explore change of instructional methods.
- Principals should be included in parts of the professional development program so they can support and encourage teachers throughout the implementation process.

Building Stakeholder Support
The introduction of a new mathematics program is likely to challenge parent and community beliefs about school mathematics and how it should be taught and learned. Teachers, principals, and other administrators must be able to explain the new program clearly and understandably to parents and concerned members of the school community-to make the case for new mathematical demands of higher education and the workplace and to explain the research basis for new approaches to teaching and learning.
Getting More Information

The seven mathematics curriculum development projects described in this document share many common philosophical and operational commitments. However, they have different ways of approaching the challenges and opportunities for improving school mathematics. Further information about each of the projects can be obtained at the following sources.

**Mathematics: Modeling Our World**  
COMAP, Inc.  
175 Middlesex Turnpike, Suite 3B  
Bedford, MA 01730  
Phone: 800-772-6627  
Publisher: COMAP, Inc.

**CME Project**  
Center for Mathematics Education  
Education Development Center, Inc.  
55 Chapel St.  
Newton, MA 02458  
Phone: 617-618-2801  
Email: cme@edc.org  
Web: [www.edc.org/cme](http://www.edc.org/cme)  
Publisher: Pearson Education

**Core-Plus Mathematics**  
Core-Plus Mathematics Project  
Western Michigan University.  
Phone: 269-387-4562  
Email: cpmp@wmich.edu  
Web: [www.wmich.edu/cpmp](http://www.wmich.edu/cpmp)  
Publisher: Glencoe/McGraw-Hill

**Interactive Mathematics Program**  
Sherry Fraser, Director  
PO Box 2891  
Sausalito, CA 94966  
Phone: 415-332-3328  
Email: imp@math.sfsu.edu  
Web: [www.mathimp.org](http://www.mathimp.org)  
[www.keypress.com/imp](http://www.keypress.com/imp)  
Publisher: Key Curriculum Press

**MATH Connections**  
It’s About Time  
Herff Jones Education Division  
84 Business Park Drive  
Armonk, NY 10504  
Phone: 914-273-2233  
Web: [www.its-about-time.com](http://www.its-about-time.com)  
Publisher: It's About Time

**SIMMS Integrated Mathematics**  
Web: [www.simms-im.com](http://www.simms-im.com)  
Publisher: Kendall/Hunt Publishing

**University of Chicago School Mathematics Project**  
6030 S. Ellis Avenue  
Chicago, IL 60637  
Phone: 773-702-1130  
E-mail: ucsmp@uchicago.edu  
Web: [socialsciences.uchicago.edu/ucsmp/](http://socialsciences.uchicago.edu/ucsmp/)  
Publisher: The Wright Group/McGraw-Hill
Development of this publication was supported in part by a grant (DRL 0808817) from the National Science Foundation. However, views expressed are those of the contributing authors, not the Foundation.