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Two High School Mathematics Curricular Paths—Which One to Take?

High schools are requiring students to complete more years of mathematics in order to graduate (Reys et al. 2007). This requirement raises several questions for schools, teachers, students, and parents. In particular, what mathematics should students study, and how should that mathematics be organized? High school mathematics programs today use two different mathematics course sequences. One sequence focuses each course on a specific subject (algebra, geometry, algebra, or precalculus), while the other integrates mathematical strands throughout each course. Choosing between subject-based and integrated course sequences stimulates discussions about—and often controversy over—which organizational choice is best and for whom.

Discussions of high school curriculum organization have often included the claim that one type is superior or inferior to the other. Web sites have been created to conduct a kind of guerrilla warfare in the mathematics community aimed at disparaging specific mathematics programs, most of which reflect a certain organizational structure. Each structure has strengths and weaknesses. We think that integrated mathematics is underrepresented in U.S. high schools. We contend that it is a viable path for a mathematics curriculum, one that deserves greater consideration.

THE SUBJECT-BASED COURSE SEQUENCE

One argument by many parents, administrators, and teachers in support of the traditional subject-based course sequence goes something like this: "This is the sequence I had when I was in school, and therefore it's what I want for my child." In other words, "I am familiar with the courses called algebra 1, geometry, and algebra 2. I may or may not have liked high school mathematics. I may or may not have been successful in high school mathematics. Regardless, I am familiar with this organizational structure of courses, and I want my child to study mathematics in the same way."

Another argument is that the subject-based organization better prepares students for college. Although this premise is debatable, the artificial separation of subject strands that occurs in some high schools also occurs at the college level, so separating courses in high school could prepare students for college-level mathematics courses. Further, the subject-based emphasis on algebraic techniques and symbol manipulation may pay dividends for students who intend to study advanced mathematics. However, the number of students engaged in advanced mathematics study at the college level has been steadily decreasing for more than twenty years (Steen 2007). The reasons are many, including students' dislike of a subject they perceive as a collection of discrete courses characterized by rigid sets of rules that lack relevance for their world (Lesh and Zawojewski 2007).

Some maintain that the traditional sequence has been successful in preparing students for college and that, as the saying goes, "If it ain't broke, don't fix it." However, others question why, if the subject-based sequence has prepared students so well for college mathematics, so few U.S. students who have successfully completed this sequence are pursuing the study of mathematics in college. Further, why has the number of remedial courses in college increased so steadily, even though states require more high school mathematics for graduation? In fact, about half of the participants in the Higher Education Research Institute Faculty Survey reported that most of their students lack basic skills required for college-level work (Lindholm et al. 2005).

Many people consider the subject-specific sequence of algebra, geometry, and algebra sacrosanct; they believe that it is the only way mathematics can be organized for instruction. In fact, the subject-specific sequence was established in the United States in the late nineteenth century to reflect recommendations spearheaded by the National Education Association (National Education Association 1894). Offering students a year of algebra followed by a year of geometry was recommended because during that era very few students completed high school, and this way they could learn some algebra and some geometry while in school. This rationale translated into geometry being sandwiched between algebra 1 and algebra 2. Although American society has changed significantly over the last one hundred years, the algebra-geometry-algebra sequence remains in place in a majority of high schools.

THE INTEGRATED MATHEMATICS SEQUENCE

Although the phrase *integrated mathematics* is defined variously (Usiskin 2003), our definition focuses on algebra, geometry, and data analysis in which connections are continuously made among these topics. The mathematics curriculum at the elementary school level is organized so that students study many strands of mathematics each year and opportunities to connect ideas across strands are embedded within curriculum materials. In this light, it makes sense to continue a structure already familiar to students. Although the elementary mathematics curriculum contains identifiable strands—such as number and operations, measurement, geometry, and data analysis—these strands are not taught separately for a year or a month. In fact, in many elementary mathematics programs, the demarcations between these strands may be unclear to the casual observer; students may be developing their understanding of number and operations as they explore measurement or data analysis. Continuing this integrated approach from elementary to secondary school mathematics programs seems a natural approach.

Real-life applications of mathematics do not artificially separate problems into discrete topics of algebra, geometry, or statistics. Most real-world problems have embedded in them a variety of very different mathematical topics. Successful problem-solving strategies require picking and choosing from one's knowledge of mathematics to produce solutions. Thus, it makes sense to organize high school mathematics in a way that will foster and develop this openness to problem solving.

If the goal of high school mathematics is to boost students' problem-solving ability, then focusing on symbol manipulation techniques and technical skills may prevent students from engaging in significant problem solving—because they will be spending most of their time developing skills. Only a small percentage of secondary school students pursue mathematical careers, but a large percentage may learn other useful skills. Integrated mathematics offers problem-solving opportunities embedded in real-world problems that many people encounter daily. For example, a business owner may never have to complete the square or solve for the complex roots of a polynomial, but she may have to decide how many items need to be sold at a particular price to turn a profit. Business owners may also need to make reliable estimates of the amount of supplies to purchase, decide on the real cost of a loan, or manage employees' work schedules. For many students, engaging in real-world problems that reflect multiple content areas (e.g., algebra, geometry, and data analysis) and that have no single algorithm may be better preparation for life after high school.

> Integrated mathematics is a viable path for a mathematics curriculum, one that deserves greater consideration

Another reason for adopting an integrated organizational structure is that nearly every country in the world except the United States uses this type of structure in its secondary school mathematics programs (Schmidt 2004). However, one needs only reflect on the lack of progress in adopting the metric system to be reminded of the difficulty of effecting change in the United States. In a country that has defied the rest of the world by ignoring the metric system, the argument to conform to how the rest of the world organizes secondary mathematics education may have little appeal.

IS THERE ONE CLEAR PATH?

What does research say about students who learn mathematics within these organizational structures? Secondary school students in the United States have consistently scored well below those in many other countries on international assessments for more than thirty years (McKnight, Crosswhite, and Dossey 1989; Schmidt et al. 1999). The overwhelming majority of U.S. students have learned their mathematics in a traditional subject-specific sequence, so, in light of these international measures of student performance, it is not possible to make a strong case for this sequence. On the other hand, there is no research regarding the influence of the integrated structure on student performance on international assessments. Although some promising results have been reported (Harwell et al. 2007; Harwell et al., forthcoming), it is too early to know how students who have been taught this way will perform on international assessments.

Many states have developed learning expectations according to subject-specific high school courses, such as algebra and geometry. Fewer states have aligned their learning expectations with integrated programs, and state assessments are greatly influenced by their learning expectations. Aligning end-of-course learning expectations with specific mathematics courses encourages schools to align their mathematics curriculum in a similar manner. The recently released *College Board Standards for College Success—Mathematics and Statistics* includes a parallel version adapted for integrated mathematics programs. This document "reflects the same content goals, but illustrates the manner in which this content might be rearranged if taught in an integrated program …" (The College Board 2007, p. 4). This model may help states and districts better accommodate whatever organizational path is taken.

Which mathematics curriculum organizational structure works best is uncertain. One undeniable fact is that the effectiveness of a particular course will not be determined solely by the organization of its content but will ultimately depend on the teacher's commitment and ability to implement the mathematics curriculum successfully and thus help students learn. A program's success will also depend on parents, administrators, and teachers working together to improve the quality of curriculum and instruction for all students. Finally, of course, whichever curricular path is taken, the students themselves must assume a major responsibility for their own learning.

So where does this leave us? Clearly there is no one right answer for every school, every teacher, or every student. To give students a choice, some high schools have chosen to implement both a subject-specific program and an integrated mathematics program. We do have choices as well as the opportunity to study the impact of alternative curricula models for organizing the mathematics curriculum. Sticking with one model when we know it is not serving all students is unprofessional. Continuing to adapt, refine, improve, and strengthen mathematics programs is the hallmark of American education and should be encouraged.

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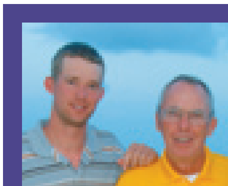

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